Lab: AP Review Sheets

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Chapter 6: Gauss's Law

Background / Summary

Gauss's Law is one of the 4 fundamental laws of electricity and magnetism. In essence, it proves that the *electric flux* through any *closed surface* is equal to the *charge within* that area divided by ε_0.

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

Using Symmetry

If you choose an area A upon which the electric field is uniform, you can remove E from the integral and the equation becomes…

$$EA = \frac{Q_{in}}{\varepsilon_o}$$

Now the hard part is usually just figuring out what Q_{in} is.

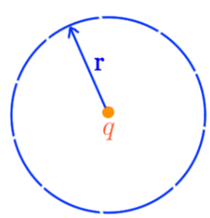
Rules for Conductors and Insulators

- For conductors, the electric field inside is zero and the charge resides on the surface
- For insulators, charge can reside throughout the volume, and the electric field inside can vary
  - need to solve for charge density

Simple Examples

"Point Charge"

$$E(4\pi r^2) = \frac{q}{\epsilon_o}$$

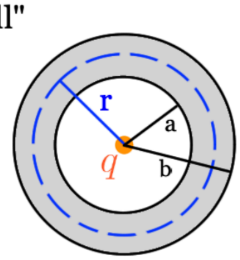


"Point charge in metal shell"

$$E(4\pi r^2) = \frac{q}{\epsilon_o} \quad \text{if r < a}$$

$$E(4\pi r^2) = 0 \quad \text{if a < r < b}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_o} \quad \text{if r > b}$$

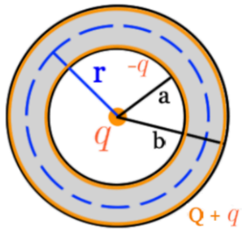


"Metal shell with net charge Q,
surrounding point charge q"

$$E(4\pi r^2) = \frac{q}{\epsilon_o} \quad \text{if } r < a$$

$$E(4\pi r^2) = 0 \quad \text{if } a < r < b$$

$$E(4\pi r^2) = \frac{q+Q}{\epsilon_o} \quad \text{if } r > b$$

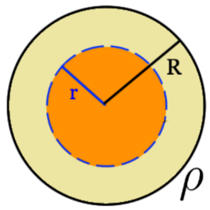


Notice how in **conducting metals**, positive charge q from the point charge attracts electrons from the conducting shell on the inside surface. This results in a -q charge on the inner surface of the shell.

"Insulating charged sphere of charge density ρ"

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_o} \quad \text{if } r < R$$

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi R^3}{\epsilon_o} \quad \text{if } r > R$$



To find the charge enclosed of an **insulated sphere**, use the relationship $\rho = \frac{Q_{total}}{V_{total}}$.

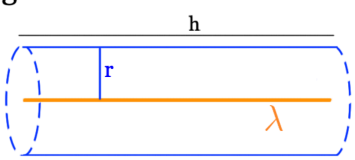
Thus: $Q = \rho V$

Notice how you multiply the charge density by the volume of your gaussian surface, so the radius varies.

Cylindrical Symmetry

"Line of charge"

$$E(2\pi r h) = \frac{\lambda h}{\epsilon_o} \quad \text{for all } r$$



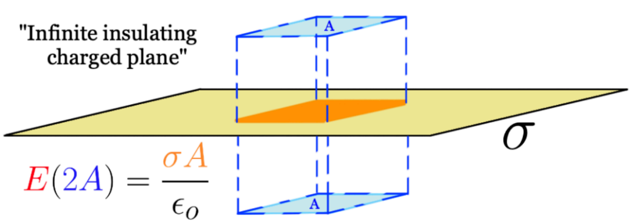
Use a cylindrical gaussian surface for a line of charge, as there is no electric flux through the caps of the cylinders.

Use the lambda relationship for Q enclosed:

$$\lambda = \frac{Q}{L}$$

Planar Symmetry

"Infinite insulating charged plane"



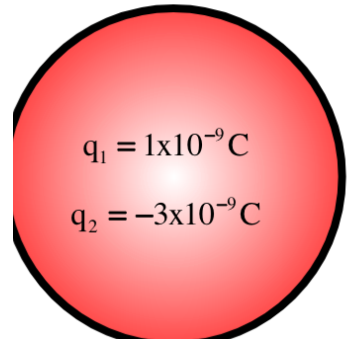
$$E(2A) = \frac{\sigma A}{\epsilon_o}$$

Use “Gaussian Plug”. Flux only piercing the two ends.

Use sigma relationship for Q enclosed:

$$\sigma = \frac{Q}{A}$$

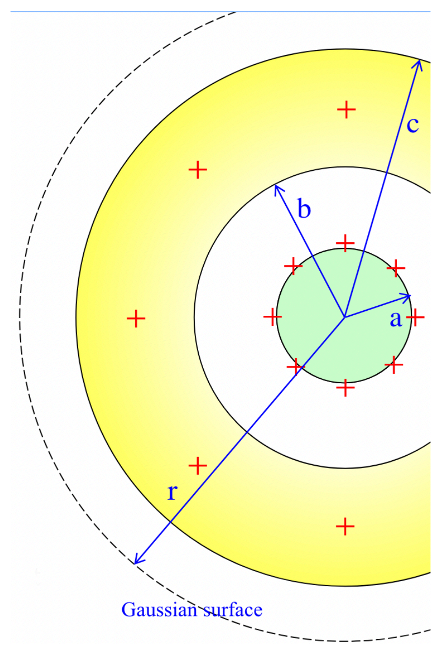
Practice Problems



1. [Easy] Assuming the two charges superimposed on the sphere are inside the sphere, determine the net electric flux through the charged sphere shown to the right.

2. [Medium]) Consider a long cylinder of radius R with charge shot uniformly throughout its volume. Assuming the volume charge density is , derive an expression of the E-fld a distance r units from the axis, where r < R. (Draw any Gaussian surface used in this problem.)

3. [Hard] A thick skinned, insulating spherical shell with inside radius b and outside radius c has a volume charge density , where the constant . At its center is a solid conducting sphere of radius a that has a net charge of Q on it.
   - Derive an expression for the magnitude of the electric field for r > c.



Solutions:

1. Q3 is outside the gaussian surface and has no effect on the electric flux. The negative sign denotes that the net electric field is inwards through the surface.

$$\Phi_E = \frac{q_{enclosed}}{\varepsilon_o} = \frac{q_1 + q_2}{\varepsilon_o} = \frac{(1x10^{-9}C) + (-3x10^{-9}C)}{8.85x10^{-12}C^2/N \bullet m^2} = -226\ N \bullet m^2/C$$

2. Because the charge density is constant, q enclosed can be found as:

$$q_{encl} = \rho V_{encl} = \rho\left[\pi r^2 (h)\right]$$

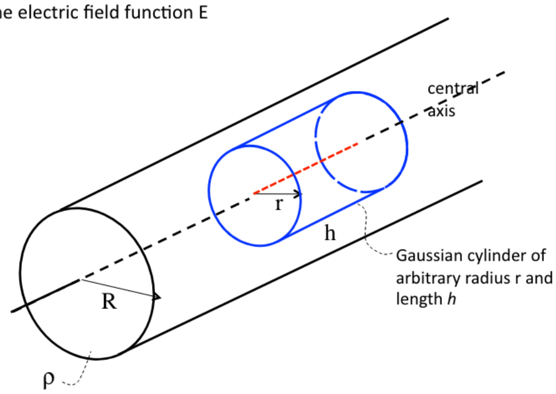
If the charge density was not constant, we'd have to solve for a differential charge density where $\rho = \frac{dQ}{dV}$, but not today!

Diagram

Finishing the problem…

A cylinder of radius R has a constant charge distribution ρ shot through it. Derive an expression for the electric field function E (r) when r <R.

For the 3-d view:



Noting that all of the flux will pass through the cylindrical part of the Gaussian surface with none through the end-caps, Gauss's Law yields:

$$\int_A \vec{E}\bullet d\vec{A} = \frac{q_{enclose}}{\varepsilon_o}$$

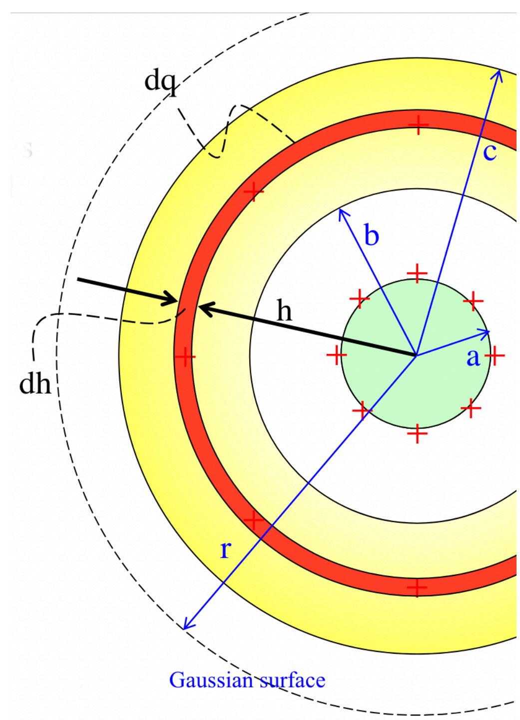
$$\Rightarrow \quad E(2\pi rh) = \frac{\rho\pi r^2 h}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\rho}{2\varepsilon_o} r$$

3. For this, we need to find total charge shot through the shell, but its not uniform. Thus we need to find a differentially thin shell of radius h with of thickness dh.

$$\rho = \frac{dq}{dV}$$

$$\Rightarrow \quad dq = \rho dV = \rho\left(4\pi h^2 dh\right) = (kh)\left(4\pi h^2 dh\right)$$



After finding what ρdV is, we integrate from h=b to c to get the charge enclosed.

$$\int_S \vec{E}\bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\Rightarrow \quad E\left(4\pi r^2\right) = \frac{Q + \int_{h=b}^{c} \rho dV}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Q + \int_{h=b}^{c} (kh)\left(4\pi h^2 dh\right)}{4\pi r^2 \varepsilon_o}$$

$$= \frac{1}{4\pi\varepsilon_o r^2}\left[Q + (4\pi k)\int_{h=b}^{c} h^3\, dh\right]$$

$$= \frac{1}{4\pi\varepsilon_o r^2}\left[Q + (4\pi k)\left(\frac{h^4}{4}\right)\Big|_{r=b}^{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_o r^2}\left[Q + (4\pi k)\left(\frac{c^4}{4} - \frac{b^4}{4}\right)\right]$$